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Extended Space-Time and the Interpretation of Wave Functions

A. I. Macfarlane

Department of Computing and Systems Analysis, Pacific Steel Limited, P.O. Box 22-201, Otahuhu, Auckland 6, New Zealand

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An extension of space and time is presented whereby a point particle can oscillate into the immediate past and future. Rapid oscillation implies that the particle is seen at, or represented over, each point in time a large number of times, each representation being associated with a different phase of the oscillation. Each cycle gives rise to an unavoidable drift from the particle's original position and hence the particle representations are scattered about the original position. Many such models of particle dynamics are possible and have implications for the interpretation of quantum mechanics and the conception of nonrealistic, nonlocal theories.

1. INTRODUCTION

Some of the conceptual difficulties arising from the quantum theoretic description of particles as quanta that interact at points while propagating in a wavelike fashion have led many authors to question the adequacy of local Euclidean space-time as starting point for kinematics; see (Bastin, 1971) for a selection of such views. In this article the relevance of macroscopic Euclidean time to the description of microscopic particles is questioned. By allowing a point particle a degree of temporal independence we present a kinematics for microscopic particles that allows

i. interaction at a point with a definite momentum,
ii. propagation of the particle without a uniquely defined position or momentum, and
iii. most convenient description of the dynamics by a complex-valued function with properties similar to those of the Schrödinger wave function.

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Thus we present a hidden variable theory based on a kinematics that is nonlocal and nonrealistic in the sense of the Einstein, Podolsky, and Rosen theorem (Clauser and Shimony, 1978; Ballentine, 1970).

The discussion in this paper is confined to the nonrelativistic kinematics of a point particle with mass and without spin.

2. THE TIME-OSCILLATING PARTICLE

The usual topological model of space and time is locally $\mathbb{R}^4$, $\mathbb{R}$ the real numbers. In the rest of this article we denote the time axis by T.

The usual (mathematical) concept of "now" or "the present" is a real number $t \in T$. Thinking of t as a Dedekind cut dramatizes how insubstantial this "present" is. We assume that such a concept has little relevance to the way a subatomic particle exists; that a measure zero "now" does not describe the time a particle "resides" in. Suppose that a particle oscillates into the near future and the near past; then, depending on the rate of oscillation it crosses "the present" a number of times. If space in the future/past is loosely connected with space "now" it can be imagined that a particle drifts from its original position with each cycle of its oscillation.

The simplest example of this oscillation in time, this temporal dynamics, is a particle with its own time t' (a real number) related to macroscopic time $t \in T$ by

$$t' = t + \Delta \sin \nu t$$

Δ and ν being dynamical properties depending on the particle. For a given time $t_0 \in T$ this particle "appears at" or "passes through" t_0 whenever $t' = t_0$ which has a set of solutions

$$R(t_0) = \{x \mid t_0 = x + \Delta \sin \nu x\}.$$

For each $x \in R(t_0)$ the particle can be thought of as being present at t_0 and x is said to correspond to a representation of the particle at t_0. The drift phenomenon separates these representations and so the particle appears at t_0 in up to card($R(t_0)$) places, where card(X) is the cardinal number of the set X. It is easily shown that for $t_0 \geqslant \pi/2\nu + \Delta$, $\mathrm{card}(R(t_0)) = 1 + [2\Delta\,\nu/\pi]$, (the square brackets being the integer part).

The more energetic the particle the more "particlelike" it becomes and the less dispersed it is. This suggests making Δ, the maximum difference from "the present" and the parameter determining the amount of drift, dependent on the energy of the particle. Thus the greater the energy the smaller Δ and the more the particle is confined to macroscopic time. If ν is identified with the frequency of the de Broglie wave of the particle then it is suggestive to set $\Delta = K \cdot \hbar / 2m$, where m is the mass and K is a constant. (If m is in kilograms and $K = 1$ m^2/sec^2 a low-energy electron has about 10^{10} representations with Δ being about 10^{-5} sec.) K is a new, unknown, constant.

When a particle interacts it does so at a point on its trajectory through space and time. In the case of absorption this destroys the generation of further representations and we experience the "collapse of ψ" phenomenon. Previous to interaction position and momentum can only be ascribed to the individual representations and none of these can be measured without interaction. When the particle interacts it does so with a given position and momentum. The cardinality of $R(t)$ gives a measure of the dispersion of momentum and position and hence a measure of the degree of "nonrealism."

Time travel is fraught with problems, and the violation of Einsteinian causality is one of them. In the model presented here a particle can interact before it is emitted and at a different place from the point of emission; if a particle is emitted at t_0, $R(t)$ can be nonempty as early as $t_0 - (\Delta - \pi/2\nu)$. $R(t)$ can also remain nonempty for a time 2Δ after a particle has been absorbed, giving "ψ" something of a life of its own.

3. INTERACTION OF THE REPRESENTATIONS

Let $\mathbf{R}(t) = \{a(t, z) | z \in R(t)\}$ be the set of representations at $t \in T$. We define the phase of a representation $a(t, z) \in \mathbf{R}(t)$ to be $\exp(i\nu z)$ and write it as $\phi(a(t, z))$ or $\phi(a)$ when the context is clear.

It is assumed that the probability of detecting a particle in a small volume V at $t_0 \in T$ depends not only on the probability of representations being present in V at t_0 but also on the phases of the representations. The phases contribute an (unnormalized) factor

$$\left| \sum_{a_k(t, z_k) \in V} [\exp(i\nu z_k)] \right|$$

(vector addition in $\mathbb{C}$, the complex numbers).

This assumes that summing the phases within V has some physical meaning. The probability that two representations coincide is vanishingly small, which implies that either phase is irrelevant or the phases of representations a small distance apart can be summed in some fashion. If we wish to describe the spatial distribution of $\mathbf{R}(t)$ by a smooth probability distribution then it is also desirable to have a smooth phase function. This phase function will have a more obvious relevance to the physical state if phases of representations have a nonlocal effect. Precisely: if $a_1, a_2 \in \mathbf{R}(t)$ and $d(a_1, a_2)$ is the distance between them, the effect of $\phi(a_2)$ on $\phi(a_1)$ at a_1 is to give a total phase

$$\phi(a_1) + \gamma(d(a_1, a_2), m) \cdot \phi(a_2)$$

where m is particle mass and γ is a smooth, positive real-valued function such that $\gamma(0, m) = 1$ and $\gamma(d, m)$ decreases as d increases. γ is thus a kinematical property associated with phase.

The γ function may seem an odd addition to the theory. Nevertheless, the reasons for its introduction would hold for all classes of theories wherein a point particle has an associated phase and interference occurs. It may of course be hidden in the measure theory at the basis of the mathematical formalism of the physical theory. In quantum mechanics γ does not appear as the complex field ψ is not taken to represent the actual location of particles; it does not measure concentration.

4. PARTICLE DYNAMICS

Let $q(a) \in \mathbb{R}^3$ be the position of a representation $a \in \mathbf{R}(t)$. $a(t, x)$ is said to vanish at t_0 if for any $\varepsilon > 0$ $(x + \varepsilon) + \Delta \sin \nu(x + \varepsilon) < t_1$ for all $t_1 > t_0$. In a similar way we can define the idea of representations "appearing."

Assume that each representation follows a course $q(a(t, z))$ that can be regarded as that of a particle subject to Brownian motion. The source of this random motion is taken to be the intrinsic looseness of space-time structure away from "the present." This can be incorporated in a fluctuating or ill-defined metric. The derivation of the Schrödinger equation from the nonlinear description of a classical particle with mass, subject to Brownian motion and with a diffusion coefficient $\hbar/2m$ and no friction (Nelson, 1966) allows us to assign a wave function ψ, a solution to Schrödinger's equation with a potential V, to each representation over its lifetime. Following E. Nelson we put $\psi_a = \exp(R_a + iS_a)$, where $R_a = \frac{1}{2} \ln \rho_a$, with ρ_a the probability density of $q(a) = q(a(t, z))$ and $\hbar$ grad $S_a = m \cdot \mathbf{v}_a$, $\mathbf{v}_a$ being the "current velocity" resulting from the influence of the potential V. By the arguments of the previous section, the state of the representations at $t \in T$ can be described by

$$\psi = \sum_{a \in \mathbf{R}(t)} \psi_a$$

The appearance and disappearance of representations means that ψ, as given, does not satisfy Schrödinger's equation over time intervals greater than 2Δ. If we replace ψ_a by $H(t_1)\cdot H(-t_2)\cdot\psi_a$, where (t_1, t_2) is the lifetime of a and $H(t)$ is the Heaviside function, ψ becomes a Schwartz distribution satisfying the Schrödinger equation except on a set of measure zero.

An alternative, though similar, approach is to regard the set of representations at any one time as an ensemble of particles subject to Brownian motion. If ρ is the mean density of the particles and $\psi=\exp(R+iS)$ with ρ, ψ, R, S, and $\mathbf{v}$ having the same relation as ρ_a, ψ_a, R_a, S_a, and $\mathbf{v}_a$ above, then ψ satisfies the Schrödinger equation (Rylov, 1971). In this case all the representations constitute a type of gas—very appropriate for the picture of an ensemble propagating but a single particle interacting.

5. FORMAL SCHEME FOR KINEMATICS

In this section we crystallize the novel aspects of the kinematics suggested in the previous sections and present a formal scheme that allows us to generalize the model of a particle given above.

Let L be a metric space, called the level space, that is a group and acts on T so that there is a map $+\colon L\times T\to T$. Furthermore there is an imbedding $j\colon T\to L$. A "space-time scheme" has the following ingredients:

i. A map $f\colon T\to L$ that factors through $S^1=\{z\,|\,z\in\mathbb{C}, |z|=1\}$, so there exists maps $\Phi(f)$ and $\Theta(f)$ such that

$$\begin{tikzcd} T \arrow[rr, "\Phi(f)"] \arrow[drr, "f"'] & & S^1 \arrow[d, "\Theta(f)"] \\ & & L \end{tikzcd}$$

commutes.

ii. A map $R(f)\colon T\to \mathrm{fte}(T)=$set of finite subsets of T. $R(f)(t_0)$ is defined as $\{t\,|\,t_0=t+f(t)\}$

iii. Let $P=\mathbb{R}^3\times L$ as a topological space; then the particle's position in P is described by a stochastic process $X\colon T\to P$ such that $\mathrm{proj}_2\cdot X=f$ (where proj_i is the projection of the ith factor of a product).

Given (i), (ii), and (iii), the description of the particle's motion is pushed back into T and $\mathbb{R}^3$ by using the map $F\colon t\mapsto\{\mathrm{proj}_i(X(t_k))|t_k\in R(f)(t)\}$ from T to $\mathrm{fte}(\mathbb{R}^3)$ together with $\{\Phi(f)(t_k)|t_k\in R(f)(t)\}=z(t)$ so giving the complete, ontological, description of the particle in T and $\mathbb{R}^3$. Replacing F and Z with what can be known gives ψ.

The maps $+$ and j are introduced for reasons of generality; there is no reason to assume that the space giving the extra degree of temporal freedom that a particle experiences is homeomorphic to T. In the account already given $L=\mathbb{R}$, $f=\Delta\sin\nu t$, and $\Phi(f)(t)=\exp i\nu t$. The particle's motion has been described by the stochastic process $X(t)$ but it can be imagined that a random motion arises from a deterministic process in an undetermined metric, that is the metric properties of P are given by a function D: $P\times P\to\mathbb{R}^+$ known only by the relationship

$$|D(x,y)-d(x,y)|<\xi(|\mathrm{proj}_2(x)-\mathrm{proj}_2(y)|)$$

where d is the ordinary product metric and $\xi(z)$ is a function that increases with z and $\xi(0)=0$. In this way uncertainty is firmly imbedded in the structure of space and time with D, not d, having physical relevance.

The space-time scheme given above can be extended to give a relativistic kinematics by expressing f, $\Phi(f)$, $R(f)$, and X as functions of proper time. In this case $T\times P$ is replaced by $M\times L$, M being $\mathbb{R}^4$ with the Minkowski metric and the domain of f, $\Phi(f)$ $R(f)$, and X is the set of orbits of the action of the proper ($\det=1$) Lorentz transformations on M [equivalently the set $\{\tau|t^2-|x|^2/c^2=\tau^2,(t,x)\in M\}$ in obvious notation]. A Lorentz transformation $\hat{\mathfrak{L}}$: $M\to M$ extends to $\mathfrak{L}$: $M\times L\to M\times L$ given by $\mathfrak{L}(x,l)=(\hat{\mathfrak{L}}(x),l/\beta)$, where $\beta=\cos\alpha$, α being the rotation about a fixed vector that gives $\hat{\mathfrak{L}}$. Thus the extension of action of the Lorentz group to $M\times L$ restricts the possible choices of L; multiplication by a real number must be possible in L and this along with other properties makes it a vector space.

As it is not our purpose to develop a relativistic theory here, we go no further than the above sketch.

6. CONCLUSION

We have presented a fragment of a hidden variable theory that approaches quantum mechanics in some of its properties. Essentially it is a variant of the stochastic interpretation of quantum mechanics (Marić and Živanović, 1976), but it differs radically in the way it involves "temporal" dynamics and so incorporates phase and in the novel aspect of representations of a particle. Furthermore, phase has a clear physical meaning and S^1 can be seen as the phase manifold for "temporal" dynamics. The theory suggests that the concept of a "nonrealistic" theory is open to further analysis. By changing the concept of a particle's time we have given a finitely nonrealistic theory, the degree of nonrealism being the upper bound of card($R(f)(t)$) in any physical configuration. Clearly stronger degrees of nonrealism include a countable range of position and momenta per particle, a continuous range, and finally "anything"; that is, any position and momentum is possible during propagation. This last case is clearly the case of maximum ignorance but is not as strong as the assertion that these parameters have no meaning until "measurement" takes place: "semantic" nonrealism.

The theory given in this paper refers only to particles with nonzero mass. Its extension to photons depends on finding a suitable Δ that does not grow too quickly with diminishing energy; longwave and hence scattering by large objects seems to imply that Δ is large. This leads to the physical system seemingly anticipating the starting up of, say, a radio wave transmitter. It is interesting to speculate on how such a phenomenon would have been interpreted without regard to temporal oscillation. The interpretation of this anticipation in the case of the electron is obscured by the extent of the phenomenon, the value of the constant K, if it is indeed a constant, and whether the number of representations is to be large, small, variable, or constant. Including spin into the theory could resolve some of these questions by altering the nature of the level space. Alternatively an attractive solution might be to replace $f\colon T \to L$ with a strictly positive function, such as $\Delta \sin^2 \nu t$ in our example [taking positive to mean $t + f(t) \geqslant t$]. In this case antiparticles can easily be conceived as having $f(t)$ replaced by $-f(t)$.

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